

Research Article

Optimization of Sun Tracking Data Handling to Improve Efficiency of PV Module

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A B S T R A C T

The procedure of tracking the sun depends on azimuth, altitude and declination angle of sun, latitude and longitude of a location and others. The orientation of sun around the earth can be tracked by using sun tracking mechanisms to collect the maximum percentage of incidents sun ray normal to the panel. This study is conducted to meet objective i.e. optimizing data handling. Three case studies have been implemented. In the first case, a computerized sun tracking device is assumed to rotate the photovoltaic panel as per the sun movement. The output is accurate, but it is expensive as typical multiple sensors, mechanical drive and microcontroller are used. In the second scenario, the solar panel is fixed at sun azimuth (ϕ) 180° and altitude 58° angle which causes more variations in the percentage of incident sun ray normal to the panel throughout the day. Whereas, in the third case, the panel is supposed to change its direction according to the 12 paths (trajectory) of sun instead of 365 paths and each path is comprised with the monthly average of the sun azimuth and altitude angle. This procedure will reduce data handling/ sun tracking cost as compared to the first case.

Keywords: Solar Energy, Sun Tracking Mechanism, Incident Sun Ray, Photovoltaic Panel

Introduction

India is among those countries where large amount of energy is produced through renewable sources. In the electricity sector, renewable energy (excluding large hydro) is accounted for 20% of the total installed power capacity (71.325 GW) as of 30 June 2018. Renewable energy resources such as tidal, wind, geothermal and other non conventional energy play constructive role to protect the environment surrounded with living entities. Owing to serious environmental issues of fossil fuels and their rapid depletion exploited the current energy situation. Therefore, proper balance between economic development, energy security and environmental issues is incontestably

essential.¹ Among all renewable energy sources, the solar energy is pure source of hygienic and constant energy to generate other forms of the energy without causing the environment. Mostly in arid zones of developing countries like India solar insolation levels are high. Therefore scientists from arid area emphasized on the solar energy to overcome continuous exploitation of conventional energy and environmental pollution. There are many fields such as solar distillation, power generation, vegetable/ fruit drying where solar energy has been used instead of fossil fuel or other conventional energy which are being depleted swiftly by humans. Commercial PV module is best to convert solar energy in to useful energy. It gives 15%-19% energy

conversion efficiency while it is being fixed. The annual solar insolation could be 10% more with the selection of optimum tilting and azimuth angle of fixed solar panel¹² and 30.79% more PV electricity is obtained with dual axis sun tracking system when it is compared with the latitude tilt fixed solar system.¹³ The two axes tracking surface with PLC control has shown better performance with an increase in the collected energy of up to 41.34% compared with the fixed surface.³ But according to study of other author,² the performance of the computerized tracking system improves by 22% more than earlier. Bentaher⁴ used a tracking system based on light dependent resistors to interact with heavy sun position data because sun trajectory angular position changes every minute in a day, every day of the month and every month of the year. Similarly, the two axis tracking of sun has been done by capturing shadow of object with web camera and converting this data into input signal of tracker's servo motor.¹⁰

On the other hand, the fixed system was described fairly simple, low-cost electromechanical set- up with low maintenance requirements with easy installation and operation except the efficiency.⁵⁻⁶ Huang⁷ has used 1A-3P (one axis three position) method to reduce cost efficiency without compromising PV system efficiency. The test result shows that the energy loss of 1A-3P PV due to misalignment of south is negligible. But according to Huang⁷ regular attention is required to change the position of PV module/ panel and also the angular deflection of incident sun rays which comes straight on the photovoltaic panel reduces its power generation. In order to handle the data with the help of PLC is very difficult and it consumes lot of power generated by PV module to run two DC motors with respect to quick changing of the sun position. When the literatures related to sun tracker were studied, it is found that the parameters such as the mechanism and its installation cost, the running cost, the efficiency, the design and the maintenance make it more complex to track the whole system.

All the above authors used PLC, electronic controller to control two DC motors for tracking altitude and surface azimuth angle of the sun position. However the sensor based tracker generates more power but it is expensive and complicated in comparison to the sensor less or fixed tracker.¹¹ Moreover, here balance between cost efficiency and efficiency of sun tracking system is required to make it acceptable which with effects tried to achieve in this study by optimizing the sun tracking data handling process.

Modeling of System

Collection of sun position (Azimuth and Altitude Angle) data. The horizontal position (azimuth angle) and height (altitude) of the sun anywhere in the world on any day can be seen on the website.⁸ An azimuth is an angular measurement in

a spherical coordinate system. The vector from an observer to a point of interest is projected perpendicularly onto a reference plane; the angle between the projected vector and a reference vector on the reference plane is called an azimuth. The data concerning to this study regarding azimuth (ϕ) of sun for each day of the year 2015 at time zone GMT+5.30 of latitude 31.3260 and longitude 75.5762 (for Jalandhar, Punjab, India) has been taken with 15minute interval time from the website⁸ to proceed further in this study. The collected data about sun horizontal position angle i.e. azimuth angle of sun for Jalandhar, Punjab, India at various time slots of a solar day length and for each day of year 2015 has been arranged and shown in Table (1). Moreover, there is not any yearly variation in the horizontal and vertical position of the sun. So the data of sun position of year 2015 would be assumed to be same for all years.

Table 1. Azimuth Angle of Sun for Jalandhar, Punjab India at various time of solar day length and on each day of year

i (Day)/ j (Time)	5:15	5:30		19:45
1	$\phi_{1,5:15}$	$\phi_{1,5:30}$		$\phi_{1,19:45}$
2	$\phi_{2,5:15}$	$\phi_{2,5:30}$		$\phi_{2,19:45}$
:	:	:	:	:
:	:	:	:	:
365	$\phi_{365,5:15}$	$\phi_{365,5:30}$		$\phi_{365,19:45}$

The row 'i' of Table (1) represents the particular day of a year and column 'j' represents the time from sunrise to sunset with fifteen minutes interval and each cell of the Table (1) is showing azimuth angle (ϕ_j) of sun for particular time and day of the year. Similarly an altitude (α) is measured in the vertical plane, between the sun's direction and the horizontal surface; in some texts this is referred to as 'elevation' or 'profile angle'. The concerning data of sun altitude angle for each day of the year at time zone GMT+5.30 of latitude 31.3260 and longitude 75.5762 (for Jalandhar, Punjab, India) has been taken with 15 minute interval from the website⁸ to proceed this study. The data of solar altitude angle has been arranged as shown in Table (2).

Table 2. Altitude Angle of sun for Jalandhar, Punjab India at various time of solar day length and on each day of year

i (Day)/ j (Time)	5 :15	5:30		19:45
1	$\alpha_{1,5:15}$	$\alpha_{1,5:30}$		$\alpha_{1,19:45}$
2	$\alpha_{2,5:15}$	$\alpha_{2,5:30}$		$\alpha_{2,19:45}$
:	:	:	:	:
:	:	:	:	:
365	$\alpha_{365,5:15}$	$\alpha_{365,5:30}$		$\alpha_{365,19:45}$

In above table (2), the row 'i' represents the particular day of a year and column 'j' represents the time from sunrise to sunset with fifteen minutes time interval and each cell is showing altitude angle (α_{ij}) of sun for particular time and day of year.

Solar Hour Angle (ω)

The solar hour angle is an expression of time, expressed in angular measurement usually degrees with respect to solar noon. At solar noon the hour angle is 0.000 degree. The time before solar noon expressed with negative degrees and the local time after solar noon expressed with positive degrees, each hour represents 15° angle. Column 'j' which represents the time of each day of year is used for the calculation and hour angle is calculated by following equation 1:

$$\omega_{ij} = \{(j - 12:00) \times 15^\circ \text{ per hour}\}$$

$$\text{for } j = 5:15, 5:30, \dots, 19:45$$

$$\text{for } i = 1, 2, 3, 4, 5, 6, \dots, 365 \quad (1)$$

Table 3. Solar hour Angle of Sun for Jalandhar, Punjab, India at time from sunrise to Sunset or solar day length

i(Day)/ j(Time)	5:15	5:30		19:45
1	$\omega_{1,5:15}$	$\omega_{1,5:30}$		$\omega_{1,17:45}$
2	$\omega_{2,5:15}$	$\omega_{2,5:30}$		$\omega_{2,17:45}$
:	:	:	:	:
:	:	:	:	:
365	$\omega_{365,5:15}$	$\omega_{365,5:30}$		$\omega_{365,17:45}$

Hence solar hour angle (ω) for a particular day time will remain constant for $i = 1, 2, 3, 4, 5, \dots, 365$.

$$\omega_{1,5:15} = \omega_{2,5:15} = \dots = \omega_{365,5:15}$$

$$\omega_{1,5:30} = \omega_{2,5:30} = \dots = \omega_{365,5:30}$$

and so on

According equation 1, solar hour angle for $j = 5:15$ and $i = 1$ would be:

$$\omega_{1,5:15} = \{(5.15 - 12.00) \times 15^\circ \text{ per hour}\}$$

$$\omega_{1,5:15} = \{(-6:45 \times 15^\circ \text{ per hour})\} = -101.25^\circ$$

$$\omega_{1,5:15} = \omega_{2,5:15} = -101.25^\circ$$

and solar hour angle for $j = 5:15$ and $i = 2$

$$\omega_{2,5:15} = \{(5.15 - 12.00) \times 15^\circ \text{ per hour}\}$$

$$\omega_{2,5:15} = \{(-6:45 \times 15^\circ \text{ per hour})\} = -101.25^\circ$$

$$\text{or } \omega_{1,5:15} = \omega_{2,5:15} = -101.25^\circ$$

Similarly other solar hour angle could be calculated for all cells of Table (3).

Latitude Angle (ϕ'')

This is the angle between a line that points from the center of the Earth to a location on the Earth's surface and a line that points from the center of the Earth to the equator. This can easily be found on a map and varies with point of location on earth. It does mean that latitude angle (ϕ'') at Jalandhar, Punjab, India is same at all time of each solar day length (from sunrise to sunset) of the year as shown in Table (4).

Table 4. Latitude Angle at Jalandhar, Punjab, India at all time from Sunrise to Sunset or Solar Day length for year

i(Day)/ j (Time)	5:15	5:30		19:45
1	$\phi''_{1,5:15}$	$\phi''_{1,5:30}$		$\phi''_{1,17:45}$
2	$\phi''_{2,5:15}$	$\phi''_{2,5:30}$		$\phi''_{2,17:45}$
:	:	:	:	:
:	:	:	:	:
365	$\phi''_{365,5:15}$	$\phi''_{365,5:30}$		$\phi''_{365,17:45}$

$$\phi''_{ij} = \text{Constant} = 31.326^\circ \}$$

$$\text{for } i = 1, 2, 3, 4, \dots, 365.$$

$$\text{for } j = 5:15, 5:30, \dots, 17:45$$

$$\text{or } \phi''_{1,5:15} = \phi''_{1,5:30} = \dots = \phi''_{1,17:45} = \phi''_{2,5:15} = \phi''_{2,5:30} = \dots = \phi''_{2,17:45} = \phi''_{ij} = 31.326^\circ$$

Declination Angle (δ)

This is the angle between the line that points towards the sun from the equator and the line which come straight out from the equator (at solar noon). North is positive and south is negative. This angle varies from +23.45 to -23.45 throughout the year, which is related to why we have seasons.

Angle of declination:

$$= \delta = 23.45 \sin \left[\frac{360}{365} (284 + n) \right]$$

Here, $n = i = \text{Days of Year Counted from 1st Jan}$:

$$\text{Angle of declination} = \delta_{ij} =$$

$$23.45 \sin \left[\frac{360}{365} (284 + i) \right]$$

$$\text{for } i = 1, 2, 3, \dots, 365$$

$$\text{for } j = 5:15, 5:30, \dots, 17:45 \quad (2)$$

The equation no. (2) has been written in tabular form as shown in table no. (5) and value of solar declination angle is constant for $j = 5:15, 5:30, \dots, 17:45$.

Table 5. Latitude angle at Jalandhar, Punjab, India at all time from sunrise to sunset or solar day length for year

i(Day)/ j (Time)	5:15	5:30		19:45
1	$\delta_{1,5:15}$	$\delta_{1,5:30}$		$\delta_{1,17:45}$
2	$\delta_{2,5:15}$	$\delta_{2,5:30}$		$\delta_{2,17:45}$
:	:	:	:	:
:	:	:	:	:
365	$\delta_{365,5:15}$	$\delta_{365,5:30}$		$\delta_{365,17:45}$

i.e. $\delta_{1,5:15}, \delta_{2,5:15}, \dots, \delta_{365,5:15} = \delta_{1,5:30}, \delta_{2,5:30}, \dots, \delta_{365,5:30} = \dots$
 $\delta_{1,17:45}, \delta_{2,17:45}, \dots, \delta_{365,17:45}$

For example for $i = 1, j = 5:15$

$$\text{Angle of declination} = \delta_{1,5:15} = 23.45 \sin \left[\frac{360}{365} (284 + i) \right]$$

$$\text{Angle of declination} = \delta_{1,5:15} = 23.45 \sin \left[\frac{360}{365} (284 + 1) \right]$$

$$\text{Angle of declination} = \delta_{1,5:15} = -23.0116^\circ$$

Similarly for $i = 1, j = 5:30$

$$\text{Angle of declination} = \delta_{1,5:30} = -23.45 \sin \left[\frac{360}{365} (284 + i) \right]$$

$$\text{Angle of declination} = \delta_{1,5:30} = -23.45 \sin \left[\frac{360}{365} (284 + 1) \right]$$

$$\text{Angle of declination} = \delta_{1,5:30} = -23.0116^\circ$$

$$\delta_{1,5:15} = \delta_{1,5:30}$$

Incident Angle (θ)

In geometric optics, the angle of incidence is the angle between a ray incident on a surface and the line perpendicular to the surface at the point of incidence, called as the normal. When the location of sun with respect to earth is at point 'P' from where sun rays are reaching to a point 'o' lying on the surface of PV panel, then the sun location is shown in polar coordinate. Hence the polar position of the point 'P' at any given time during the solar day length of each month of the year is expressed with the sun azimuth angle ' ϕ_{ij} ', altitude angle ' α_{ij} ' and radial distance ' r_{ij} ' from sun to the point 'o'.

i.e. Polar coordinate of 'P' = ($r_{ij}, \phi_{ij}, \alpha_{ij}$)

But if, at same moment, the normal to the point 'o' from point 'Q' is slightly deflected from the incident sun radiation (from point 'P'), then polar location of the point 'Q' will be at radius ' r'_{ij} ' from point 'Q' to point 'o' on PV panel, azimuth angle ' ϕ'_{ij} ' and altitude angle ' α'_{ij} '.

i.e. Polar coordinates of point 'Q' = ($r'_{ij}, \phi'_{ij}, \alpha'_{ij}$)

Hence the incident angle θ_{ij} is angle between the line

joining point P($r_{ij}, \phi_{ij}, \alpha_{ij}$) with point 'o' and the line joining point Q($r'_{ij}, \phi'_{ij}, \alpha'_{ij}$) with point 'o' as shown in figure (26).

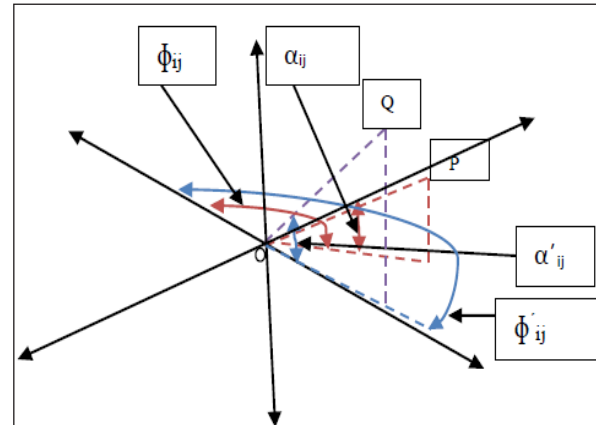


Figure 26

The calculation of the Cartesian on x_{ij}, y_{ij}, z_{ij} axis from both point 'P' and 'Q' is required to determine the solar incident angle θ_{ij} at any moment of the year. Following equations are used to precede it. Cartesian values for point P($r_{ij}, \phi_{ij}, \alpha_{ij}$) are

$$x_{ij} = r_{ij} \times \cos(360 - \phi_{ij}) \times \cos(\alpha_{ij}) \quad (3)$$

$$y_{ij} = r_{ij} \times \sin(360 - \phi_{ij}) \times \cos(\alpha_{ij}) \quad (4)$$

$$z_{ij} = r_{ij} \times \sin(\alpha_{ij}) \quad (5)$$

Similarly Cartesian for the point Q($r'_{ij}, \phi'_{ij}, \alpha'_{ij}$)

$$x'_{ij} = r'_{ij} \times \cos(\alpha'_{ij}) \cos(360 - \phi'_{ij}) \quad (6)$$

$$y'_{ij} = r'_{ij} \times \cos(\alpha'_{ij}) \sin(360 - \phi'_{ij}) \quad (7)$$

$$z'_{ij} = r'_{ij} \times \sin(\alpha'_{ij}) \quad (8)$$

Let's assume radius ' r'_{ij} ' is equal to radius ' r_{ij} ' as the sun will be at same radial distance from point 'o' at particular moment when sun either is at the point 'P' or 'Q' i.e.

$$r_{ij} = r'_{ij}$$

Suppose 'L' is straight line between points 'P' and 'Q' instead of arc as radial distance from point 'P', 'Q' to point 'o' is very large. The value of 'L' is calculated by filling all Cartesians values taken from equations no.(3)-(8) in following straight line formula or in equation no. (9)

$$L_{ij} = \sqrt{(x_{ij} - x'_{ij})^2 + (y_{ij} - y'_{ij})^2 + (z_{ij} - z'_{ij})^2} \quad (9)$$

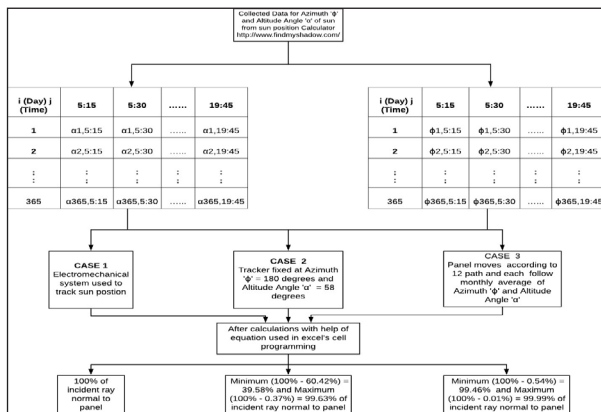
The radial distance r_{ij} and r'_{ij} from the point 'o' on PV panel to point 'P' and 'Q' respectively are same. Therefore the arc length and incident angle i.e. $\theta_{ij}, \theta'_{ij}$ for both radius will be equal

Incident angle =

$$= \theta_{ij} = \theta'_{ij} = \frac{\text{Arc length}}{r_{ij}} = \frac{(\text{Arc length})'}{r'_{ij}} \quad (10)$$

Thus the sun incident ray angles for various moments of the day of each month of the year have been calculated from the given data regarding azimuth, altitude, latitude and hour angles on web site.⁸

Methodology to Optimize Sun Tracking Data Handling



Methodology of this Study

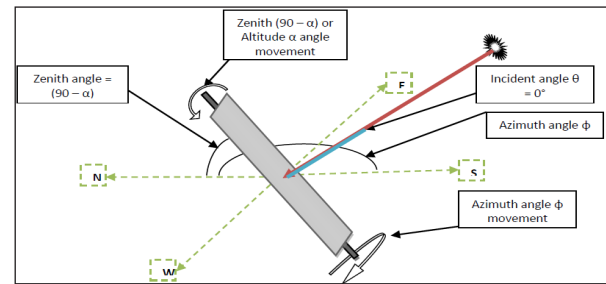
Optimization of Sun Tracking Data Handling (Performance and Evaluation)

In regards to this, the collected data of sun positions (azimuth and altitude) of latitude 31.3260° (Jalandhar, Punjab, India) with time interval of 15 minutes for each day of the year is huge. Therefore handling such data for tracking the movement of sun throughout the day is very difficult and obviously costs a fortune. Hence, it is recommendable to simplify the data handling optimization to avoid overhead cost. The optimization of the sun position data (azimuth ϕ and altitude α) that given in table 1 and 2 is done by comparing the recommended data handling method with the two axis tracking system and fixed PV system.

- (Case 1) Two axis movement of collector/ PV module according to the real sun position throughout day with any electromechanical mechanism.^{3,6}
- (Case 2) South facing collector/ PV module is fixed at 32° zenith angle or 58° (90°-32°) altitude angle.^{2,3}
- (Case 3) Recommended movement of collector or PV module throughout the month according to the monthly average of azimuth (ϕ) and altitude (α) angle which reduces from 365 sun trajectories to 12 sun trajectories for a year as shown in table 6 and table 7 respectively.

Case 1

The data for Case 1 regarding azimuth (ϕ)₁ and altitude (α)₁ angle of PV module for latitude 31.3260° (Jalandhar, Punjab, India) has been taken from the website⁸ to conclude this study. The two axes, sun tracking system enables to generate more energy as the solar panel is always perpendicular to the incident sun ray.



CASE I. Panel moving on 365 Trajectories Comprised actual Sun Azimuth ϕ and Altitude ϕ Angle

Sun' ray normal to panel for Case 1 = $(I_{ij})_1 = I_{ij} \cos(\theta_{ij})_1$ (11)

For case 1, sun tracking system $(\theta_{ij})_1 = 0^\circ$

tracking system $(\theta_{ij})_1 = 0^\circ$, $(I_{ij})_1 = I_{ij}$

Hence two axis tracking system uses 100% incident sun rays for power generation, The collected data from the website⁸ in tabular form (as in Table 1 and 2) is very large. So to simplify this study, the considered data of azimuth and altitude for selected time like 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year has to be analyzed. The changing of azimuth and altitude angle of the PV module according to the collected data of selected time from the website⁸ has been shown in Figure 1 and Figure 2, respectively.

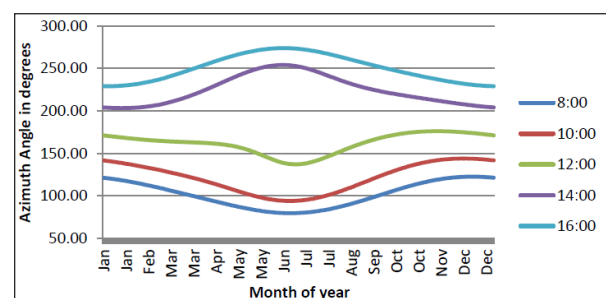


Figure 1. Azimuth Angle for Case I at 15 minutes Interval for each day of the year

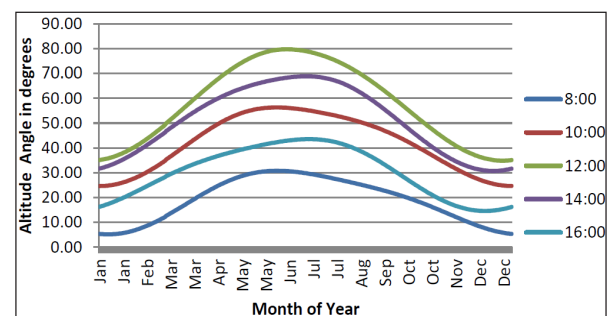
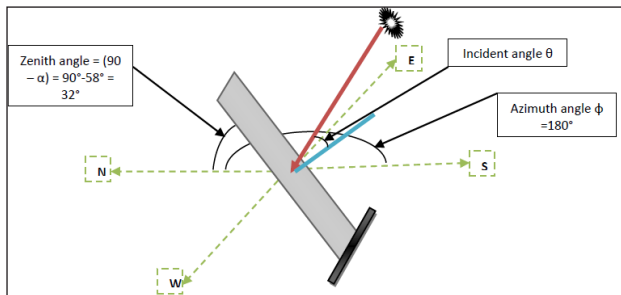


Figure 2. Altitude Angle for Case I at 15 minutes Interval for each day of the year

Case 2

In case 2 the PV module is fixed at position facing south or on 180° azimuth angle and on 58° altitude angle.

or $(\phi_{ij})_2 = 180^\circ$ and $(\alpha_{ij})_2 = 58^\circ$



CASE 2. Fixed Panel at Azimuth $\phi = 180^\circ$ and Altitude $\alpha = 58^\circ$

The Equations 3-10 has been developed to calculate theoretical value of the sun incident ray angle $(\theta_{ij})_2$ for case 2. The long calculations are required to extract theoretical values of the sun incident ray angle $(\theta_{ij})_2$ for latitude 31.3260° (Jalandhar, Punjab, India), with interval of 15 minutes for each day of year. It is possible with the help of excel cell programming. The change of values of the sun incident ray angle $(\theta_{ij})_2$ at 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year has been represented in Figure 3.

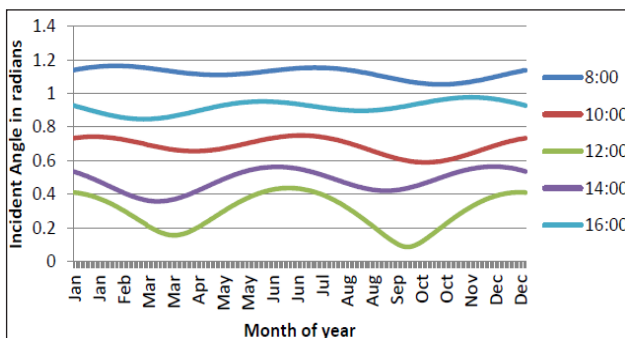


Figure 3. Incident Angle in Case 2 at 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year

Sun's ray normal to panel for

$$\text{Case 2} = (I_{ij})_2 = (I_{ij}) \cos(\theta_{ij})_2 \quad (12)$$

The equation no. 12 is used to calculate the incident sun ray normal to panel and these values are taken at 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year which have been represented in Figure 4.

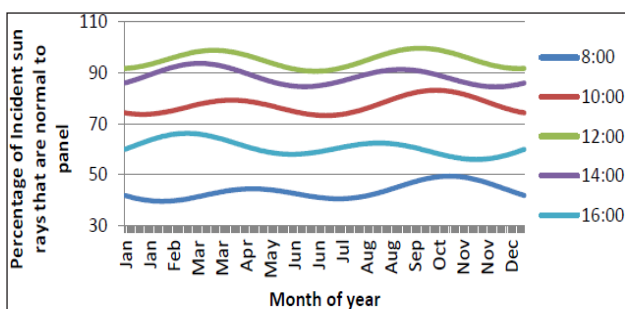
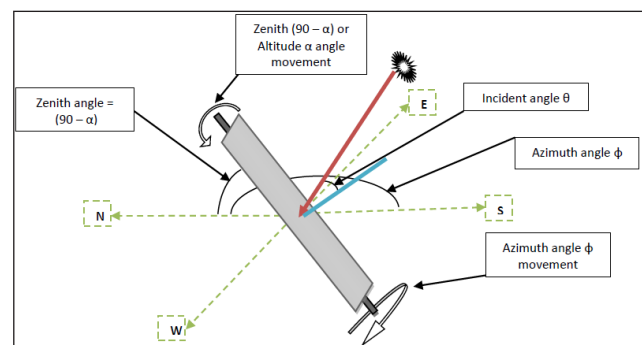


Figure 4. Percentage of Incident Sun Rays Normal to the Panel in Case 2 at 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year

Case 3

The Case 3 is recommended method to optimize the handling of huge data of sun position. This method is helpful to summarize the number of sun positions from 85727 to 708. Consequently, this optimization of data or angles (azimuth ϕ_{ij} and altitude α_{ij}) of PV module reduces 365 sun trajectories (each sun trajectory for each day of the year) to 12 sun trajectory data (each sun trajectory for each month of the year). Each sun trajectory comprises with altitude angles for various azimuth angles. As mentioned earlier in Case 3, the movement of collector or PV panel throughout the month is done according to the monthly average of azimuth (ϕ) and altitude (α) angle of same month. These monthly average of azimuth (ϕ) and altitude (α) angle have been calculated as shown in table 6 and 7 respectively and in another words it is extract of table 1, and table 2.



CASE 3. Panel Moving on 12 Trajectory Comprised Monthly Average of sun Azimuth ϕ and Altitude α

As earlier discussed, this study is conducted at selected times of the day i.e. at 8:00, 10:00, 12:00, 14:00 and 16:00, for each month of the year. This method of the optimization of data handling to track sun position may be helpful to bridge the gap between the cost efficiency and efficiency of the solar energy collector. According to Case 3, recommended method, the variation of azimuth (ϕ_{ij3}) and altitude (α_{ij3}) throughout the day of each month of the year to optimize sun position data represented in Figure 5 and 6.

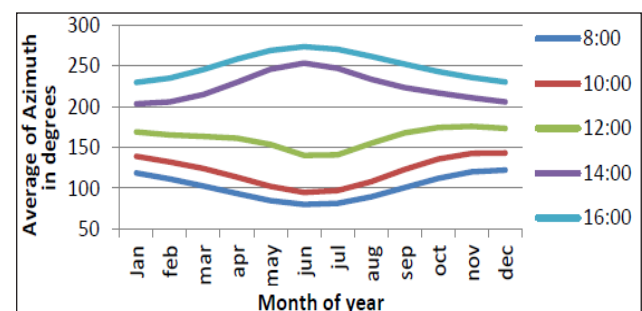


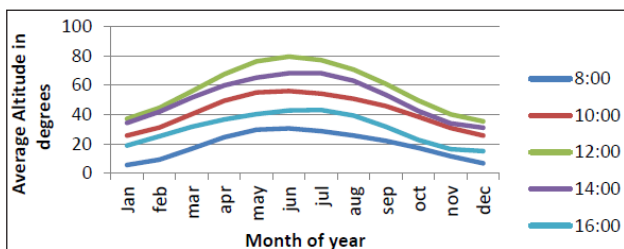
Figure 5. Monthly Average Azimuth Angle for Case 3 at 15 Minutes Interval for each day of the year

Table 6. Monthly Average of Sun Azimuth Angle (ϕ) for Jalandhar, Punjab India at Various time of Solar day of each Month of the year

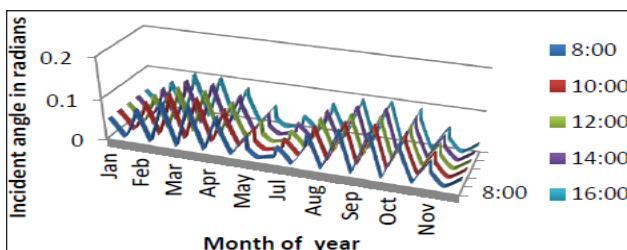
k (Month)/ j (Time)	5:15	5:30		19:45
Jan	$\frac{\sum_{i=0}^{31} \phi(i, 5:15)}{31}$	$\frac{\sum_{i=0}^{31} \phi(i, 5:30)}{31}$		$\frac{\sum_{i=32}^{59} \phi(i, 19:45)}{28}$
Feb	$\frac{\sum_{i=32}^{59} \phi(i, 5:15)}{28}$	$\frac{\sum_{i=32}^{59} \phi(i, 5:30)}{28}$		$\frac{\sum_{i=32}^{59} \alpha(i, 19:45)}{28}$
:	:	:	:	:
Dec	$\frac{\sum_{i=335}^{365} \phi(i, 5:15)}{31}$	$\frac{\sum_{i=335}^{365} \alpha(i, 5:30)}{31}$		$\frac{\sum_{i=335}^{365} \alpha(i, 19:45)}{31}$

Table 7. Monthly Average Altitude Angle (α) of Sun for Jalandhar, Punjab India at various time of Solar day of each Month of the year

k (Month)/ j (Time)	5:15	5:30		19:45
Jan	$\frac{\sum_{i=0}^{31} \alpha(i, 5:15)}{31}$	$\frac{\sum_{i=0}^{31} \alpha(i, 5:30)}{31}$		$\frac{\sum_{i=0}^{31} \alpha(i, 19:45)}{31}$
feb	$\frac{\sum_{i=32}^{59} \alpha(i, 5:15)}{28}$	$\frac{\sum_{i=32}^{59} \alpha(i, 5:30)}{28}$		$\frac{\sum_{i=32}^{59} \alpha(i, 19:45)}{28}$
:	:	:	:	:
Dec	$\frac{\sum_{i=335}^{365} \alpha(i, 5:15)}{31}$	$\frac{\sum_{i=335}^{365} \alpha(i, 5:30)}{31}$		$\frac{\sum_{i=335}^{365} \alpha(i, 19:45)}{31}$

**Figure 6. Monthly Average Altitude Angle for Case 3 at 15 minutes Interval for each day of the year**

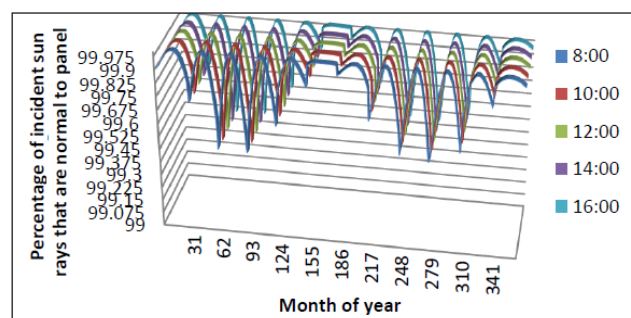
As the Equations 3-10 have been used to calculate sun incident ray angle in Case 2, similarly the sun incident ray angles for Case 3 ($\theta_{ij,3}$) have been calculated with help of excel's cell programming. The variation of sun incident ray angle ($\theta_{ij,3}$) at 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year has been represented in Figure 7.

**Figure 7. Incident Angle in Case 2 at 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year**

Sun rays normal to panel for:

$$\text{Case 3} = (I_{ij})_3 = (I_{ij}) \cos(\theta_{ij,3}) \quad (13)$$

The equation no. 13 is used to calculate the values of the incident sun rays normal to the panel and these values are taken at 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year. It has represented in Figure 8.

**Figure 8. Percentage of Incident Sun rays normal to the Panel in Case 3 at 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year**

Analysis

The comparison of all cases based on following two factors is done to conclude the study

- Due to loss of the incident sun rays, rays are not being converted into electricity through PV cell

- To calculate variation in the percentage of incident sun rays those are normal to the panel or is being converted into electricity through PV cell throughout the day and year.

The correlation and least R^2 method is best to project total percentage of variation and incident sun rays normal to the panel for whole year at 8:00, 10:00, 12:00, 14:00 and 16:00. It would be helpful to conclude the best among the three cases considered and discussed in this study.

Due to loss of the Incident Sun ray in all cases, rays are not being Converted into Electricity

Data analysis of all cases is required to calculate the percentage of incident sun rays which are not being used to convert solar energy into electrical energy and this analysis is conducted for a series of time 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year.

Total percentage of incident Sun rays on surface of panel/ earth for a series of time 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year = I_{ij}

Total percentage of incident Sun rays that are normal to the panel in any of the above considered cases for a series of time 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year = $(I_{ij})_k$

Loss of the incident sun rays since they are not being converted into electricity:

$$= I_{ij} - (I_{ij})_k \quad (14)$$

Case 1 Analysis

As mentioned earlier, electromechanical system, sensors or closed loop control system etc are used in this case to retain zero degrees incident angle on panel or to keep panel in the position when 100% incident sun rays that are normal to the panel throughout each day of the year. Hence the total area under the curve of the percentage of incident sun rays which is normal to the panel w.r.t. the day of the year at 8:00 would be 100×365 days of year i.e. 36500 in Figure 9. Similarly, the area under the curve for 10:00, 12:00, 14:00 and 16:00 time series has shown in Table 8.

Case 2 Analysis

Similarly the various mathematic model / equation for different trend lines and R^2 values for all time series (i.e. 8:00, 10:00, 12:00, 14:00 and 16:00) has been taken from the percentage of incident sun rays normal to the panel in Case 2 as shown in Table 9-13. It was cleared from Table 9-13, that the logarithmic trend line model/ equation is suitable to represent the Case 2's data for the percentage of sun ray normal to panel. Its graphical form was shown in the Figure 10-14. The total sum of the percentage of incident sun ray normal to the panel during 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year for Case 2 has to be calculated similar to the Case 1 in Table 8.

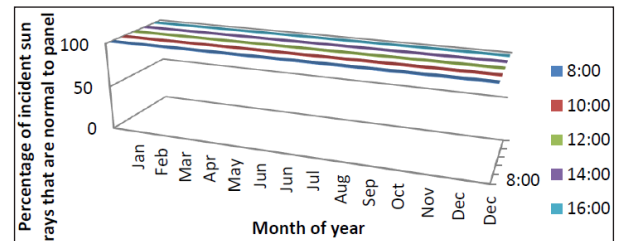


Figure 9. Percentage of Incident Sun rays normal to the Panel in Case 1 at 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year

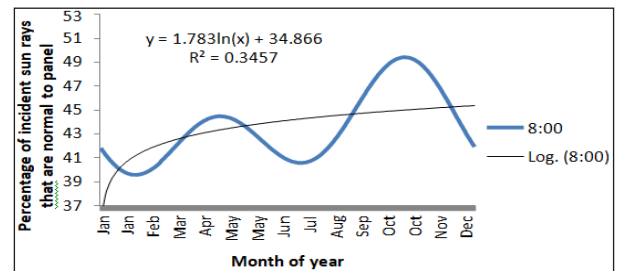


Figure 10. Equation and R^2 for Logarithmic Trend line for Case 2

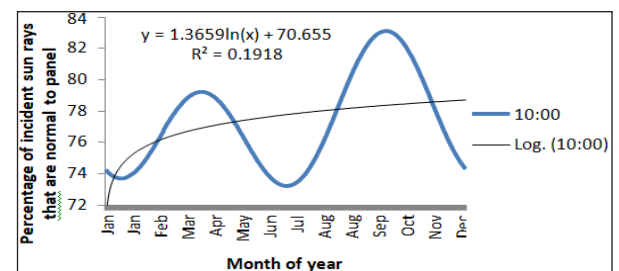


Figure 11. Equation and R^2 for Logarithmic trend line for Case 2

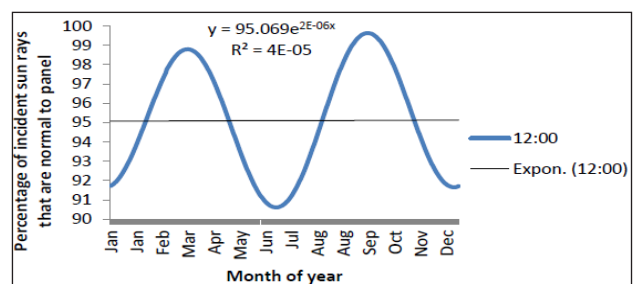


Figure 12. Equation and R^2 for Exponential Trend line for Case 2

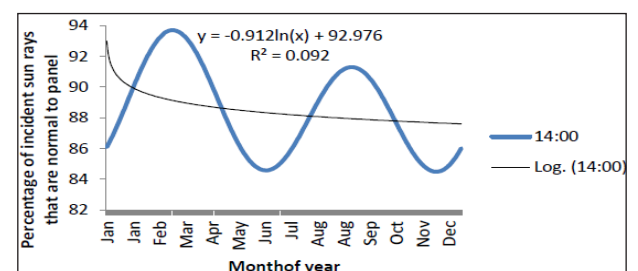


Figure 13. Equation and R^2 for Logarithmic Trend line for Case 2

Table 8. Area under the Suitable Mathematic Models/ Equations for Trend Lines in Case 1

S. No.	Series of Time in Figure 9	Type of Trend line	Suitable Mathematic Model/ Equation	Area Under Curve
1.	08:00	Constant = 100	$y = \text{Constant } \%$	36500
2.	10:00	Constant = 100	$y = \text{Constant } \%$	36500
3.	12:00	Constant = 100	$y = \text{Constant } \%$	36500
4.	14:00	Constant = 100	$y = \text{Constant } \%$	36500
5.	16:00	Constant = 100	$y = \text{Constant } \%$	36500
Total sum of percentage of incident sun rays throughout the year that are normal to panel during 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year 2015				182500

Table 9. Various Model/ Equation with R^2 Value at 8:00 for Figure 4 in Case 2

S.No.	Type of Trend line	Suitable Equation at time 8:00 for Figure 4 (Case 2)	R^2 Value
1.	Six order Polynomial	$y = 1E-12x^6 - 2E-09x^5 + 7E-07x^4 - 0.000x^3 + 0.013x^2 - 0.496x + 44.48$	0.967
2.	Five order Polynomial	$y = -2E-10x^5 + 1E-07x^4 - 3E-05x^3 + 0.003x^2 - 0.121x + 41.12$	0.877
3.	Four order Polynomial	$y = -2E-08x^4 + 1E-05x^3 - 0.002x^2 + 0.212x + 36.97$	0.709
4.	Three order Polynomial	$y = -5E-07x^3 + 0.000x^2 - 0.017x + 41.23$	0.488
5.	Two order Polynomial	$y = -1E-06x^2 + 0.019x + 40.10$	0.468
6.	Exponential	$y = 40.21e^{0.000x}$	0.472
7.	Linear	$y = 0.019x + 40.13$	0.468
8.	Logarithmic	$y = 1.783 \ln(x) + 34.86$	0.345

Table 10. Various Model/ Equation with R^2 value at 10:00 for Figure 4 in Case 2

S.No.	Type of Trend line	Suitable Equation at time 10:00 for Figure 4 (Case 2)	R^2 Value
1.	Six order Polynomial	$y = 2E-12x^6 - 2E-09x^5 + 9E-07x^4 - 0.000x^3 + 0.014x^2 - 0.404x + 76.39$	0.983
2.	Five order Polynomial	$y = -6E-11x^5 + 3E-08x^4 - 1E-06x^3 - 0.001x^2 + 0.195x + 71.02$	0.767
3.	Four order Polynomial	$y = -3E-08x^4 + 2E-05x^3 - 0.004x^2 + 0.333x + 69.31$	0.740
4.	Three order Polynomial	$y = -6E-07x^3 + 0.000x^2 - 0.025x + 75.94$	0.232
5.	Two order Polynomial	$y = -2E-05x^2 + 0.021x + 74.52$	0.202
6.	Exponential	$y = 75.04e^{0.000x}$	0.194
7.	Linear	$y = 0.012x + 75.03$	0.196
8.	Logarithmic	$y = 1.365 \ln(x) + 70.65$	0.191

Table 11. Various Model/ Equation with R^2 value at 12:00 for Figure 4 in Case 2

S.No.	Type of Trend line	Suitable Equation at time 12:00 for Figure 4 (Case 2)	R^2 Value
1.	Six order Polynomial	$y = 2E-12x^6 - 2E-09x^5 + 9E-07x^4 - 0.000x^3 + 0.010x^2 - 0.135x + 92.39$	0.980
2.	Five order Polynomial	$y = 7E-11x^5 - 9E-08x^4 + 4E-05x^3 - 0.007x^2 + 0.51x + 86.61$	0.707
3.	Four order Polynomial	$y = -3E-08x^4 + 2E-05x^3 - 0.004x^2 + 0.354x + 88.55$	0.669
4.	Three order Polynomial	$y = -5E-07x^3 + 0.000x^2 - 0.029x + 95.64$	0.037
5.	Two order Polynomial	$y = -3E-05x^2 + 0.011x + 94.39$	0.011
6.	Exponential	$y = 95.06e^{2E-06x}$	4×10^{-5}
7.	Linear	$y = 0.000x + 95.10$	6×10^{-5}
8.	Logarithmic	$y = 0.315 \ln(x) + 93.59$	0.011

Table 12.Various Model/ Equation with R² value at 14:00 for Figure 4 in Case 2

S.No.	Type of Trend line	Suitable Equation at time 14:00 for Figure 4 (Case 2)	R ² Value
1.	Six order Polynomial	$y = 2E-12x^6 - 1E-09x^5 + 5E-07x^4 - 7E-05x^3 + 0.001x^2 + 0.195x + 84.97$	0.953
2.	Five order Polynomial	$y = 2E-10x^5 - 2E-07x^4 + 7E-05x^3 - 0.011x^2 + 0.660x + 80.80$	0.813
3.	Four order Polynomial	$y = -2E-08x^4 + 1E-05x^3 - 0.003x^2 + 0.252x + 85.87$	0.559
4.	Three order Polynomial	$y = -3E-07x^3 + 0.000x^2 - 0.027x + 91.07$	0.224
5.	Two order Polynomial	$y = -2E-05x^2 - 0.005x + 90.39$	0,216
6.	Exponential	$y = 90.80e^{-1E-0x}$	0.211
7.	Linear	$y = -0.012x + 90.82$	0.212
8.	Logarithmic	$y = -0.91\ln(x) + 92.97$	0.092

Table 13.Various Model/ Equation with R² value at 16:00 for Figure 4 in Case 2

S.No.	Type of Trend line	Suitable Equation at time 16:00 for Figure 4 (Case 2)	R ² Value
1.	Six order Polynomial	$y = 5E-13x^6 - 3E-10x^5 + 2E-08x^4 + 3E-05x^3 - 0.006x^2 + 0.423x + 57.74$	0.941
2.	Five order Polynomial	$y = 2E-10x^5 - 2E-07x^4 + 7E-05x^3 - 0.010x^2 + 0.579x + 56.34$	0.925
3.	Four order Polynomial	$y = -7E-09x^4 + 5E-06x^3 - 0.001x^2 + 0.082x + 62.53$	0.555
4.	Three order Polynomial	$y = 1E-09x^3 + 5E-06x^2 - 0.021x + 64.45$	0.509
5.	Two order Polynomial	$y = 6E-06x^2 - 0.022x + 64.46$	0.509
6.	Exponential	$y = 64.35e^{-3E-0x}$	0.510
7.	Linear	$y = -0.019x + 64.33$	0.509
8.	Logarithmic	$y = -1.71\ln(x) + 69.11$	0.320

Table 14.Area under the Suitable Mathematic Models/ Equations for Trend lines in Case 2

S.No.	Series of Time in Figure 4	Type of Trend line	Suitable Mathematic Model/ Equation	Area under the Curve by Integrating the Equations no. 15-19
1.	08:00	Logarithmic	$y = 1.783 \ln(x) + 34.86$	15879.65
2.	10:00	Logarithmic	$y = 1.365 \ln(x) + 70.65$	28159.22
3.	12:00	Exponential	$y = 95.06e^{2E-06x}$	34614.51
4.	14:00	Logarithmic	$y = -0.91 \ln(x) + 92.97$	32212.67
5.	16:00	Logarithmic	$y = -1.71 \ln(x) + 69.11$	22096.06
Total sum of percentage of incident sun rays throughout the year that are normal to the panel during 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year				132962.11

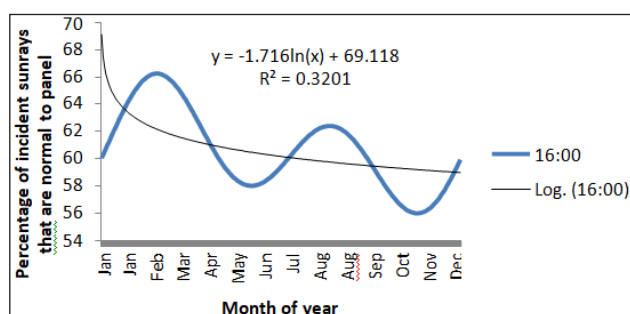


Figure 14.Equation and R² for logarithmic trend line for Case 2

After comparison of the eight different trend lines shown in Table 9-13, the logarithmic equations are suitable for all series of time i.e. 8:00, 10:00, 14:00 and 16:00 except 12:00 since the value of R² is least among the other equation / model of trend lines. Similarly the exponential model / equation correlate with the curve at time 12:00. Further the calculation of the area under the curve for all suitable equations have been calculated by integrating the Equations no. 15-19 with limits from 1st to 365th day of the year as shown in table 14.

For series of time 8:00 $y = 1.783\ln(x) + 34.86$ (15)

For series of time 10:00 $y = 1.365\ln(x) + 70.65$ (16)

For series of time 12:00 $y = 95.06e^{2E-06x}$ (17)

For series of time 14:00 $y = -0.91\ln(x) + 92.97$ (18)

For series of time 16:00 $y = -1.71\ln(x) + 69.11$ (19)

Case 3 Analysis

The values of incident angle at various time (8:00, 10:00, 12:00, 14:00 and 16:00) for a particular day in Case 3 remains same as shown in Figure 7. Hence the value of the percentage of incident sun rays normal to the panel at various times for that particular day remains same as shown in Figure 8. The equations that are suitable to the curves (Figure 8) would also be same since the variation of the percentage of sun rays normal to the panel at 8:00, 10:00, 12:00, 14:00 and 16:00 for the year is similar as represented in Figure 8. Further the various types of trend line equations and R^2 values of the curves for the all series of time (8:00, 10:00, 12:00, 14:00 and 16:00) in Figure 8, have shown in Figure 15-22.

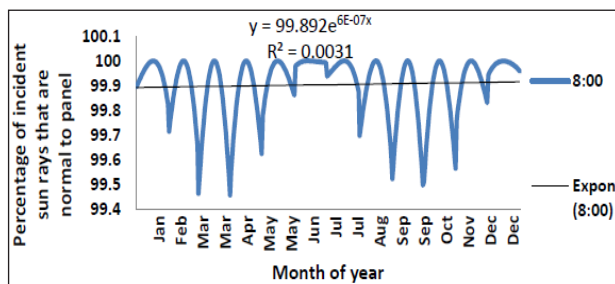


Figure 15. Equation and R^2 for Exponential Trend line for Case 3

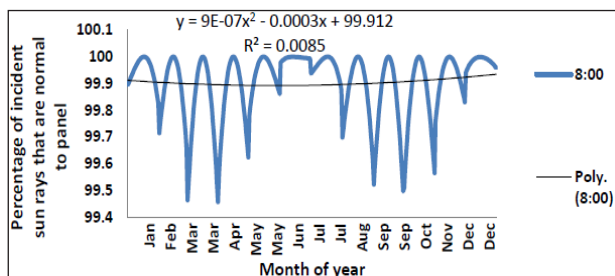


Figure 16. Equation and R^2 for Second order Polynomial Trend line for Case 3

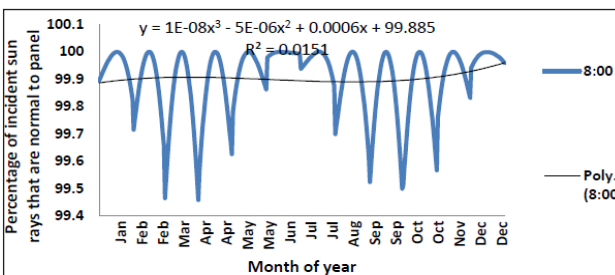


Figure 17. Equation and R^2 for third order Polynomial Trend line for Case 3

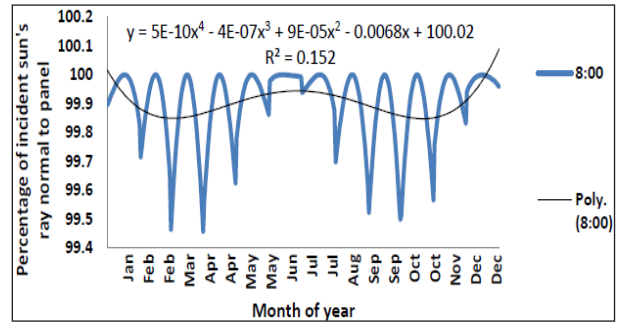


Figure 18. Equation and R^2 for Fourth order Polynomial Trend line for Case 3

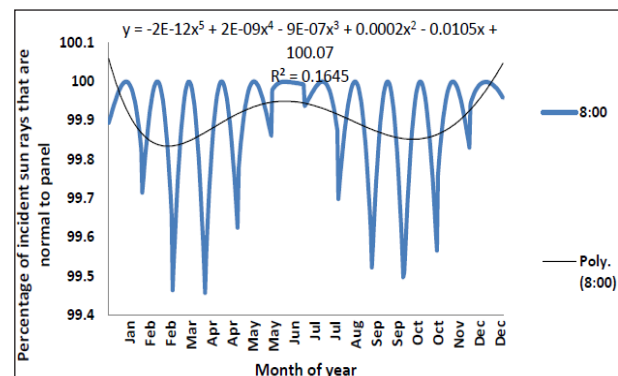


Figure 19. Equation and R^2 for Fifth order polynomial Trend line for Case 3

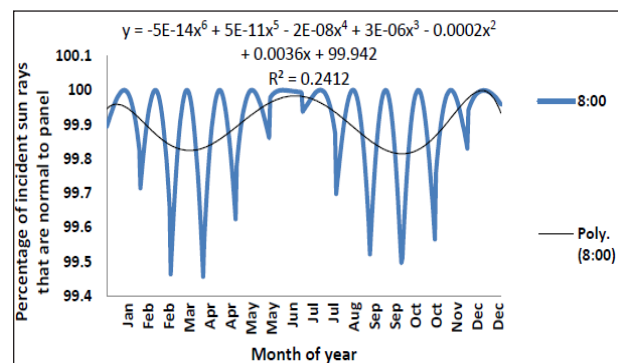


Figure 20. Equation and R^2 for Sixth order Polynomial Trend line for Case 3

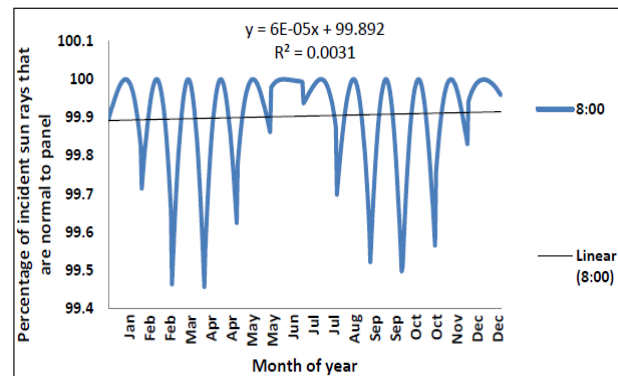


Figure 21. Equation and R^2 for Linear trend line for Case 3

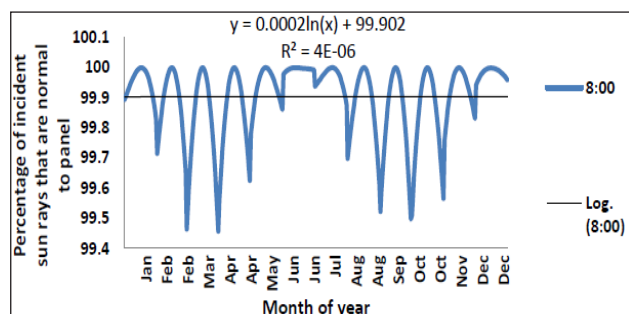


Figure 22. Equation and R^2 for Logarithmic Trend line for Case 3

It is observed from the Table 15, that logarithmic trend line has at least R^2 ($4E-06$) value in comparison to other types of the trend line. Hence logarithmic trend line equation no. 20 in Case 3 is suitable for all series of time in Figure 8. The values of the percentage of incident sun rays normal to the panel for various days of the year have been calculated with logarithmic equation with least variation with respect to the mean. The area under the curve for Figures 15-22, is calculated to distinguish the small dissimilarity in the percentage of incident sun rays that are normal to the panel for the considered series of time (8:00, 10:00, 12:00, 14:00

and 16:00). Further the area under the curves is calculated by integrating the logarithmic trend line Equation No. 20 with limits from 1st day to 365th and was shown in Table 16.

$$y = 0.000\ln(x) + 99.90 \quad (20)$$

As generally when the normal on panel makes zero degree angles with incident sun rays then 100% incident sun's rays are utilized to convert solar energy into electrical energy. Total percentage of incident sun rays that are utilized for the three considered cases in this study has been shown in Table 17.

Variation of the percentage of incident sun rays that are normal to the panel:-

There are two types of variations in the percentage of incident sun rays that are normal to the panel.

- Variation in the percentage of incident sun rays that are normal to the panel throughout each day of the year.
- Variation in the percentage of incident sun rays that are normal to the panel throughout the year.

Analysis has been done on above mentioned variations for time series 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year.

Table 15. Model / Equation Suitable for Series 8:00, 10:00, 12:00, 14:00 and 16:00 for Figure 8 in Case 3

S.No.	Type of Trend line	Suitable Equation for series of time 8:00, 10:00, 12:00, 14:00 and 16:00 for Figure 8 (Case 3)	R-Square Value
1.	Six order Polynomial	$y = -5E-14x^6 + 5E-11x^5 - 2E-08x^4 + 3E-06x^3 - 0.000x^2 + 0.003x + 99.94$	0.164
2.	Five order Polynomial	$y = -2E-12x^5 + 2E-09x^4 - 9E-07x^3 + 0.000x^2 - 0.010x + 100.0$	0.164
3.	Four order Polynomial	$y = 5E-10x^4 - 4E-07x^3 + 9E-05x^2 - 0.006x + 100.0$	0.152
4.	Three order Polynomial	$y = 1E-08x^3 - 5E-06x^2 + 0.000x + 99.88$	0.015
5.	Two order Polynomial	$y = 9E-07x^2 - 0.000x + 99.91$	0.008
6.	Exponential	$y = 99.89e^{6E-07x}$	0.003
7.	Linear	$y = 6E-05x + 99.89$	0.003
8.	Logarithmic	$y = 0.000\ln(x) + 99.90$	0.000004

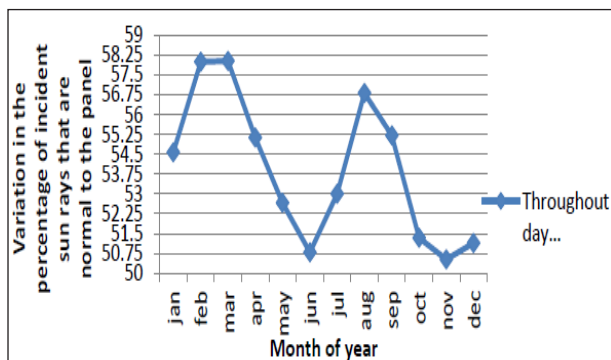
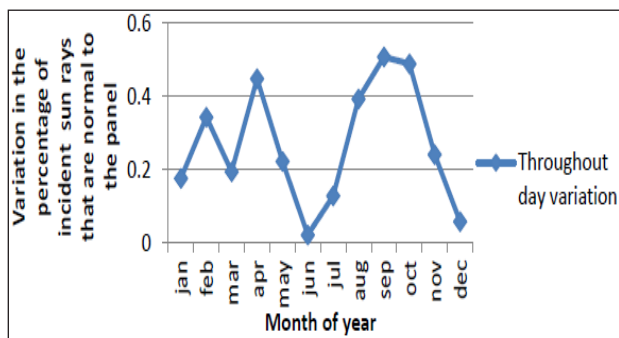
Table 16. Area Under the Suitable Mathematic Models/ Equations for Trend lines in Case 3

S.No.	Series of Time in Figure 8	Type of Trend line	Suitable Mathematic Model/ Equation	Area Under the Curve by Integrating Equation no. 15
1.	08:00	Logarithmic	$y = 0.000\ln(x) + 99.90$	36363.6
2.	10:00	Logarithmic	$y = 0.000\ln(x) + 99.90$	36363.6
3.	12:00	Logarithmic	$y = 0.000\ln(x) + 99.90$	36363.6
4.	14:00	Logarithmic	$y = 0.000\ln(x) + 99.90$	36363.6
5.	16:00	Logarithmic	$y = 0.000\ln(x) + 99.90$	36363.6
Total sum of percentage of incident sun rays throughout the year that are normal to the panel during 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year				181818

Table 17. Total Sum of the Percentage of Incident Sun rays lost throughout the year that are not normal to the Panel during 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of year

S.No.	Various conditions of this study	Total sum of Percentage of Incident sun rays throughout the year that are normal to the panel during 8:00, 10:00, 12:00, 14:00 and 16:00 of each day of the year	Total percentage of incident sun rays utilized	Loss of incident sun rays
1	CASE 1	$36500 \times 5 = 182500$	100%	0%
2	CASE 2	132962.11	72.85%	27.15%
3	CASE 3	181818	99.63%	0.37%

The variation throughout the day of the year for Case 1: The sun position in form of azimuth and altitude angle is supposed to track with the help of sensitive sensor and electro-mechanical system. Consequently the PV panel uses 100% incident sun rays throughout the day and throughout the year to convert solar energy into electrical energy. Therefore there is no or zero (0%) percentage in the variation of incident sun rays that are normal to the PV panel has been noticed. Similarly in Case 2, the variation throughout the day of the percentage of incident sun ray normal to the panel has been shown in the Figure 23. It is observed from Figure 23, that the maximum value of the variation of the percentage of incident sun rays normal to the panel throughout the day is up to 58.02381% of incident sun rays.

**Figure 23. Variation in the Percentage of Incident Sun rays normal to the panel throughout the day in Case 2****Figure 24. Variation in the Percentage of Incident Sun Rays normal to the panel throughout the day in Case 3**

For Case 3 the variation in the percentage of incident sun rays normal to the panel throughout the day has been shown in Figure 24. It is observed that the maximum value of the variation in the percentage of incident sun rays normal to the panel throughout the day is up to 0.50806% of incident sun rays.

As mentioned earlier, the variation from the actual incident sun rays on the surface of the earth or panel in Case 1 throughout the year is zero percentage of the incident sun rays that are normal to the panel since PV panel moving according to actual position of sun. Similarly, in Case 2, the variation in the percentage of the incident sun rays that are normal to panel does exist due to seasonal changes occur in India as shown in the Figure 25 and it is observed that the maximum range of the variation goes up to 10.25828% of incident sun rays that strikes the surface of the earth or panel.

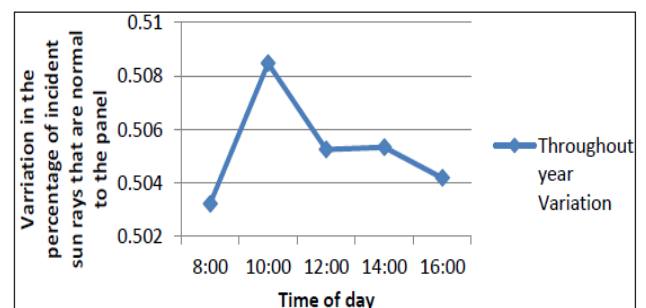
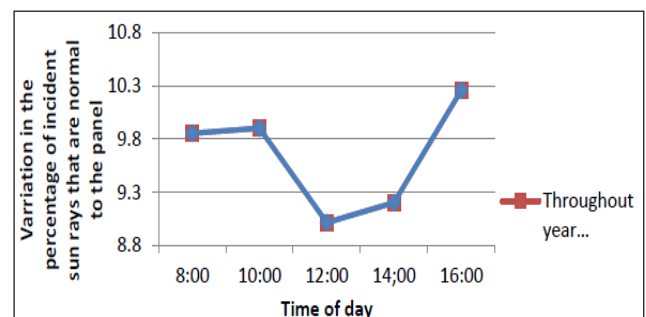
**Figure 26. Variation in the Percentage of Incident Sun Rays normal to the Panel throughout year in Case 3****Figure 25. Variation in the Percentage of incident sun rays normal to the panel throughout year in Case 2**

Table 18. Comparison of the Maximum Variation in the Percentage of Incident sun rays normal to the Panel or Utilized sun rays

CASE	Maximum variation in the percentage of incident sun rays normal to the panel or utilized sun rays throughout the day	Maximum variation in the percentage of incident sun rays normal to the panel or utilized sun rays throughout the year
Case 1	0 %	0 %
Case 2	58.02381 %	10.25828 %
Case 3	0.50806 %	0.508476 %

In third case, PV panel moves according to the monthly average of the sun's azimuth and altitude angles. The variation in the percentage of incident sun rays that are normal to the panel for case 3 is represented in the figure 26. It is clearly visible from the Figure 26 that the maximum range of the percentage of the incident sun rays goes up to 0.508476%.

Comparison of the maximum variation throughout the year and day for the three considered cases of this study has been shown in the Table 18.

Conclusion

This study has been conducted to compare

- The loss of incident sun rays or the rays that are not normal to the panel or not being utilized to convert solar energy into electrical energy.
- The variation in the percentage of incident sun rays that are normal to the panel throughout the day and year in three different sun tracking case study mechanisms of this study.

As mentioned in the Table 17 the 100% of incident sun rays that are normal to the panel in the Case 1 has been used to convert solar energy into electrical energy since the position of the panel is being changed with respect to the position of sun. It is possible only with the electromechanical control system therefore loss of the percentage of incident sun rays or the sun rays that are not normal to panel is Zero. But, tracking system of case 1 costs more as compare to other cases because of the sensitive, typical and costly electromechanical system is required.

Similarly from the Table 17, 72.85% and 99.63% of incident sun rays has been utilized or 27.15%, 0.37% of incident sun rays have been lost for Case 2 and 3 respectively. The tracking cost for Case 3 is insignificantly high than Case 2 but it is significantly less than Case 1. Case 3 tracking method loses negligible percentage (0.37%) of incident sun rays.

After comparison of the maximum value of the variation in the percentage of incident sun rays that are normal to the panel, the maximum value of the variation in the percentage of incident sun rays due to the variation of incident angle throughout the day and year has been concluded zero

percent in Case 1 as shown in the Table 18. The natural and seasonal variation in the sun radiation's intensity does not effect on the maximum value of the variation in the percentage of incident sun rays that are normal to panel as shown in Table 18. Similarly from Table 18, the maximum value of the variation in the percentage of incident sun rays that are normal to the panel throughout the day and year for Case 2 is 58.02381 %, 10.25828 % respectively which is very high as compare to Case 3 and 1. This variation does effect on the efficiency and performance of the PV panel and grid system. But the variation throughout the day and year for Case 3 is 0.50806 % and 0.508476 % respectively which is insignificantly more than Case 1 and significantly lesser than Case 2. Finally on the basis of efficiency and cost of the sun tracking method, the Case 3 has been concluded as the best mechanism as it is able to utilize more percentage of incident sun rays at cost which is lesser than Case 1 with lesser variation in the percentage of incident sun rays that are normal to the panel throughout day and year.

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